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TECHNICAL MEMORANDUM

ACOUSTIC INTERACTION IN A SPHERICAL DOME/ARRAY

by

W. C. Moyer and J. W. Duran

Submitted to
Commander, Naval Ship Systems Command
Department of the Navy
Washington, D. C. 20360
Attention: Code 00V1E

19 July 1968

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ABSTRACT

A procedure is described which permits the computation of the radiation characteristics of a transducer array given the electrical inputs to the transducer elements. The effects of transducer element interactions (acoustic coupling) and dome-transducer interactions are included in the computation.

The dome-transducer configuration chosen for this study is a rectangular array of circular elements mounted in a spherical baffle. This array is surrounded by a concentric, homogeneous dome. Numerical results include (a) radiation loadings on the transducer elements, (b) response (head velocity) of each element to prescribed electrical inputs and (c) farfield beam patterns for the dome/array. The results indicate that element interactions and dome/array interactions can affect element response to the electrical input and thereby degrade transmit performance of the transducer array.

TABLE OF CONTENTS

LIST OF FIGURES

1. INTRODUCTION

The effects of acoustic coupling among transducer elements on the performance of large sonar transducer arrays operating at low frequencies have been the subject of extensive research. In independent studies, the effect of a dome on nearfield and farfield radiation characteristics of such arrays has been investigated assuming negligible coupling effects among the elements. It has been found that the presence of a dome can significantly alter the radiation loading on an active array relative to the loading in the absence of the dome. The purpose of this memorandum is to illustrate a technique for determining the effects of acoustic coupling, both among transducer elements and between the dome and array, on array performance.

When a single transducer element is radiating into a free field, the radiation impedance seen by the element is essentially ρc, the characteristic impedance of the surrounding medium. If a specified array of elements is considered, the radiation loading on any one element depends on several factors, including element location in the array, array configuration (cylindrical, spherical, etc.), beam steering/tilt angle, and frequency. When the array is surrounded by a dome, the element radiation loadings are modified further. The radiation loading on any single element can determine the response (head velocity) of that element, assuming a known driving voltage or current. Since the element loadings can vary considerably in the array, the velocity distribution of the array and, consequently, the radiation characteristics of the dome/array can be affected compared to the ideal case for which an element response is uniquely determined by that element's applied electrical signal.

To analyze element response in the dome/array environment and subsequently compute dome/array radiation characteristics, one must have (1) a dome/array model which can treat radiation due to an arbitrary number of elements having arbitrary head velocities, and (2) a transducer element model which yields response as a function of radiation loading for prescribed input voltage or current. The procedure is demonstrated in this memorandum through utilization of a spherical array surrounded by a concentric shell and an assumed transducer element model.

2. BACKGROUND

Transducer element modeling has progressed in stages from crude lumped parameter models to more sophisticated ones in which components of the element are treated as continuous media*. As one goal, these models seek to relate realistically the response of an element to a prescribed electrical signal as a function of radiation loading. Often these models are represented by equivalent circuits. In this form, the input variables, voltage and current, can be related to the output variables, head velocity and radiation force, by a pair of linear independent equations. The coefficients in these equations are obtained through mathematical and experimental analysis of the element.

The element modeling has been accompanied by much analytical and experimental effort to determine the effect of the array environment (radiation loading) on element response**. If we consider an array of elements and assume the ith element has a head velocity v_i and as a result produces a sound pressure field p_i, then a force is created on each of the neighboring elements. In particular, the force F_{ij} on the jth element is

$$F_{ij} = \int_{S_j} p_i \, dS_j \quad , \tag{1}$$

where S_j is the area of the jth element. If F_{ij} is normalized by v_i then we define the interaction coefficient Z_{ij} between the ith and jth pistons as

*D. G. Carson, G. E. Martin, et. al., of the Transducer Division, NUWC have been principal contributors in this area.

**Among others, the NUWC group and D. T. Porter of USN/USL have been active in this area.

$$Z_{ij} = \frac{F_{ij}}{v_i} \tag{2}$$

with $Z_{ij} = Z_{ji}$.

The total force F_j on the jth element due to N active elements in the array is*

$$F_j = \sum_{i=1}^{N} F_{ij} = \sum_{i=1}^{N} Z_{ij}\, v_i \quad , \tag{3}$$

with $Z_{jj}\, v_j$ the radiation force on the jth element due to its own motion. The total radiation impedance Z_j^{rad} seen by the jth element is given as

$$Z_j^{rad} = \sum_{i=1}^{N} Z_{ij} \frac{v_i}{v_j} \quad . \tag{4}$$

If the internal impedance of the element is Z_{oc} (the same for all elements in the array), then the head velocity v_i can be related (see Ref. 1) to the current I_i (or voltage) applied to the ith element by

$$I_i = \gamma(Z_i^{rad} + Z_{oc})v_i \tag{5}$$

where γ is a complex constant dependent on the element component materials, dimensions, static biases, etc., and the operating frequency. Note from Eq. (5) that, in general, the head velocity of the ith element depends on the head velocities of all the elements, i.e., the currents and head velocities are related by a set of simultaneous equations. In matrix notation Eq. (5) is represented as

*Interaction effects due to nonactive elements are not considered here, although they can be included in the analysis.

$$\{I_i\} = [\alpha_{ij}]\{v_i\} \quad , \tag{6}$$

with $\alpha_{ij} = \gamma Z_{ij}$, $i \neq j$ and $\alpha_{ii} = \gamma(Z_{ii} + Z_{oc})$. A similar relation exists for voltage controlled elements.

The above relation permits the computation of a set of head velocities for a prescribed set of currents, assuming the Z_{ij} can be computed. For the computation of the Z_{ij}, a mathematical model is required which simulates the dome/array of interest. The array model must allow the computation of the sound pressure produced at any point on the array by an element having arbitrary location in the array and arbitrary head velocity. This model is exercised between all element pairs to obtain the set of Z_{ij}. (Symmetry considerations often allow a reduction in the required number of computations.) A set of head velocities is then determined from Eq. (6) for known $\{I_i\}$, Z_{oc}, and γ. This set of head velocities is utilized in the dome/array model to compute nearfield and farfield sound pressure levels, beam patterns, etc. Descriptions of several dome/array models which can be used in such a series of computations are given in Ref. 2.

3. TECHNICAL DESCRIPTION

The dome/array model chosen for this study consists of a rectangular array of close-packed, circular elements on a spherical baffle. The baffle is surrounded by a concentric shell as shown in Fig. 1. Radiation characteristics for this model, assuming element velocities independent of radiation loading, have been studied in Ref. 3.

The analysis is initiated by considering the sound pressure field produced by a single circular piston or element radiating with harmonic time dependence. As shown in Ref. 3, the sound pressure field p_1 between the array and dome is given by

$$p_1 = \sum_{m=o}^{\infty} [A_m j_m(kr) + B_m n_m(kr)] \; P_m(\cos\psi) \; e^{-i\omega t} \quad , \qquad (7)$$

and the sound pressure exterior to the dome, p_2, is given by

$$p_2 = \sum_{m=o}^{\infty} C_m h_m(kr) \; P_m(\cos\psi) \; e^{-i\omega t} \quad . \qquad (8)$$

In the above expressions, $j_m(kr)$, $n_m(kr)$, and $h_m(kr)$ are the mth order spherical Bessel, Neumann, and Hankel functions respectively, k is the wave number, $P_m(\cos\psi)$ is the mth order Legendre polynomial, ψ is the polar angle measured from an axis through the center of the element (the problem is axisymmetric), and ω is the angular frequency. A_m, B_m, and C_m are constants determined by the boundary conditions. The boundary conditions are defined by specifying the normal particle velocity of the fluid to be zero everywhere on the sphere, except for the piston, which has a uniform velocity $V_o e^{-i\omega t}$, and requiring the normal particle velocity at fluid-shell interfaces to be continuous.

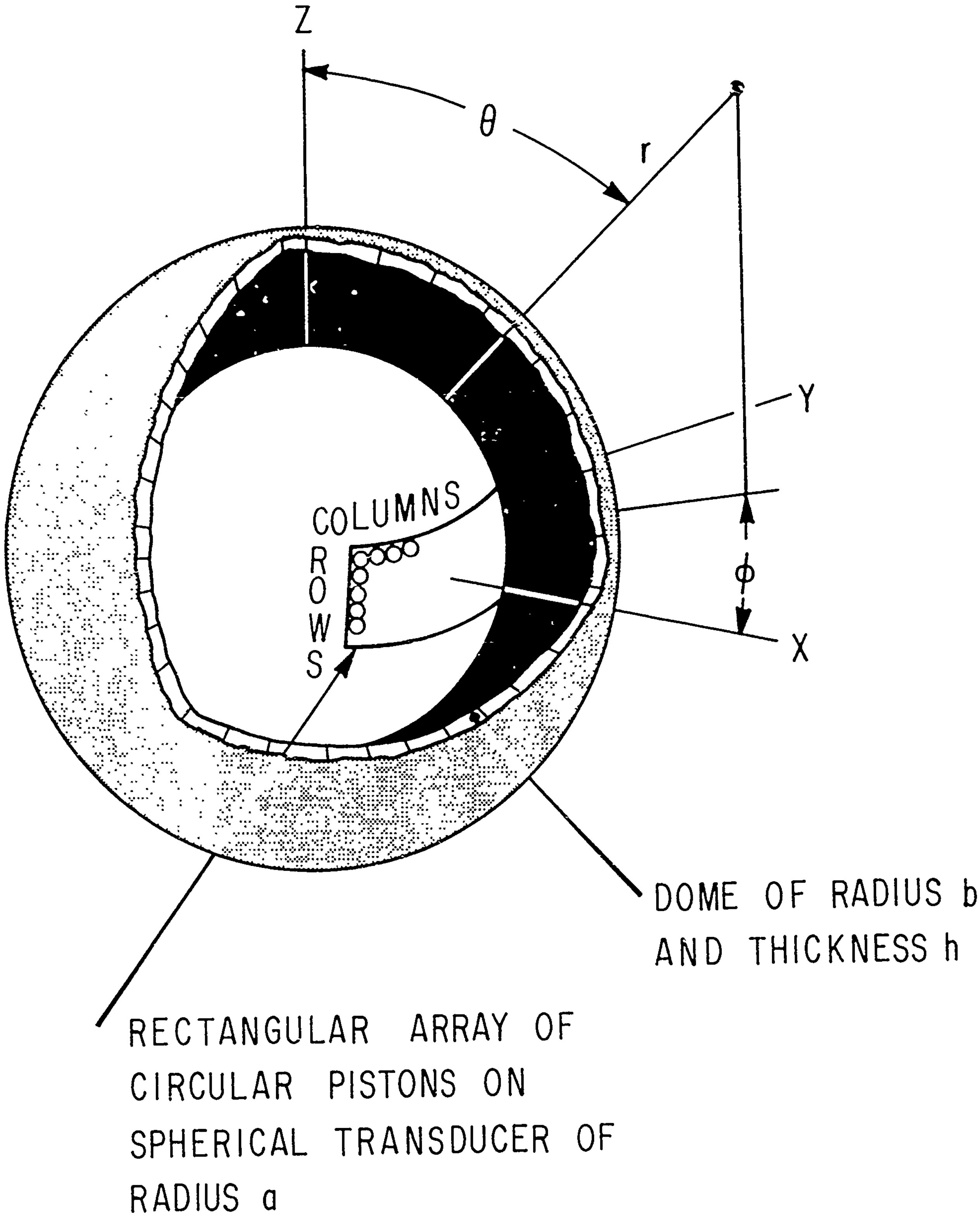

FIG. I ARRAY ON SPHERICAL TRANSDUCER WITHIN CONCENTRIC SPHERICAL DOME

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Three basic steps are involved in obtaining radiation characteristics of the dome/array from the known electrical inputs to the transducer elements. These are:

1. The loading produced by any element on the remaining elements in the array is obtained by integrating Eq. (7), evaluated on the array surface, over each element in the array for all pairs of elements as in Eq. (1). These loadings are then converted to Z_{ij} as in Eq. (2).
2. The set of head velocities is obtained by solving the set of equations given by Eq. (5) for known input currents (or voltages), element characteristics, and the Z_{ij} obtained under (1) above.
3. Equation (7) is evaluated for each element using the head velocity computed in (2) above, and the resulting element pressure fields are superimposed to obtain the sound pressure field for the array between the array and dome. The sound pressure field exterior to the dome is obtained as a superposition of the individual element pressures given by Eq. (8).

The first step, determining the Z_{ij}, is straightforward. As shown in the appendix, the sound pressure field produced by a single piston has the form

$$p_1 = V_o \frac{i\rho c}{2} \sum_n \Gamma_n \left[P_{n-1}(\cos\alpha) - P_{n+1}(\cos\alpha) \right] P_n(\cos\psi) \qquad (9)$$

when evaluated on the array surface. In the above equation, the Γ_n are constants depending on the dome/array dimensions, dome material properties, and frequency, and α is the angular half-width of the element (if the element radius is a_o and the array radius is a, then $\alpha = \arctan \frac{a_o}{a}$). To obtain the loading on the jth element in the array, separated from the ith element by an angle θ, we integrate Eq. (9) over the jth element and obtain

$$F_{ij} = V_o \pi a^2 \rho c \sum_{n=o}^{\infty} \Gamma_n \frac{[P_{n-1}(\cos\alpha) - P_{n+1}(\cos\alpha)]^2}{2n+1} P_n(\cos\theta). \quad (10)$$

F_{ij} will be normalized by $\pi a_o^2 \rho c$ so that

$$Z_{ij} = \frac{a^2}{a_o^2} \sum_{n=o}^{\infty} \Gamma_n \frac{[P_{n-1}(\cos\alpha) - P_{n+1}(\cos\alpha)]^2}{2n+1} P_n(\cos\theta) \quad . \quad (11)$$

The Z_{ij} are computed for all pairs of active elements in the array.

The next step in the series of computations is the determination of the set of element head velocities. For these computations, Z^{rad} for each element and Z_{oc} must be determined. Z_{oc} is obtained from the transducer element model. An example of the type of element considered in this memorandum is shown in Fig. 2. Along with a cross section view, a circuit schematic is shown. The components are assumed to be one-dimensional media. Wave propagation in the media is described by linear elasticity theory. The components of the element must satisfy boundary conditions at their juncture with other components. For example, forces and velocities must be continuous at the boundaries of each component. For the passive components (head mass, tail mass, etc.), displacements satisfy the scalar wave equation, but for the active component (ceramic), a mechanical strain component due to the applied electrical signal must be considered. Solutions to the governing differential equations can be obtained by usual techniques. Due to the coupling of the element components at the boundaries (e.g. stress rod - tail mass interface), the equations describing component motion are coupled. The derivation

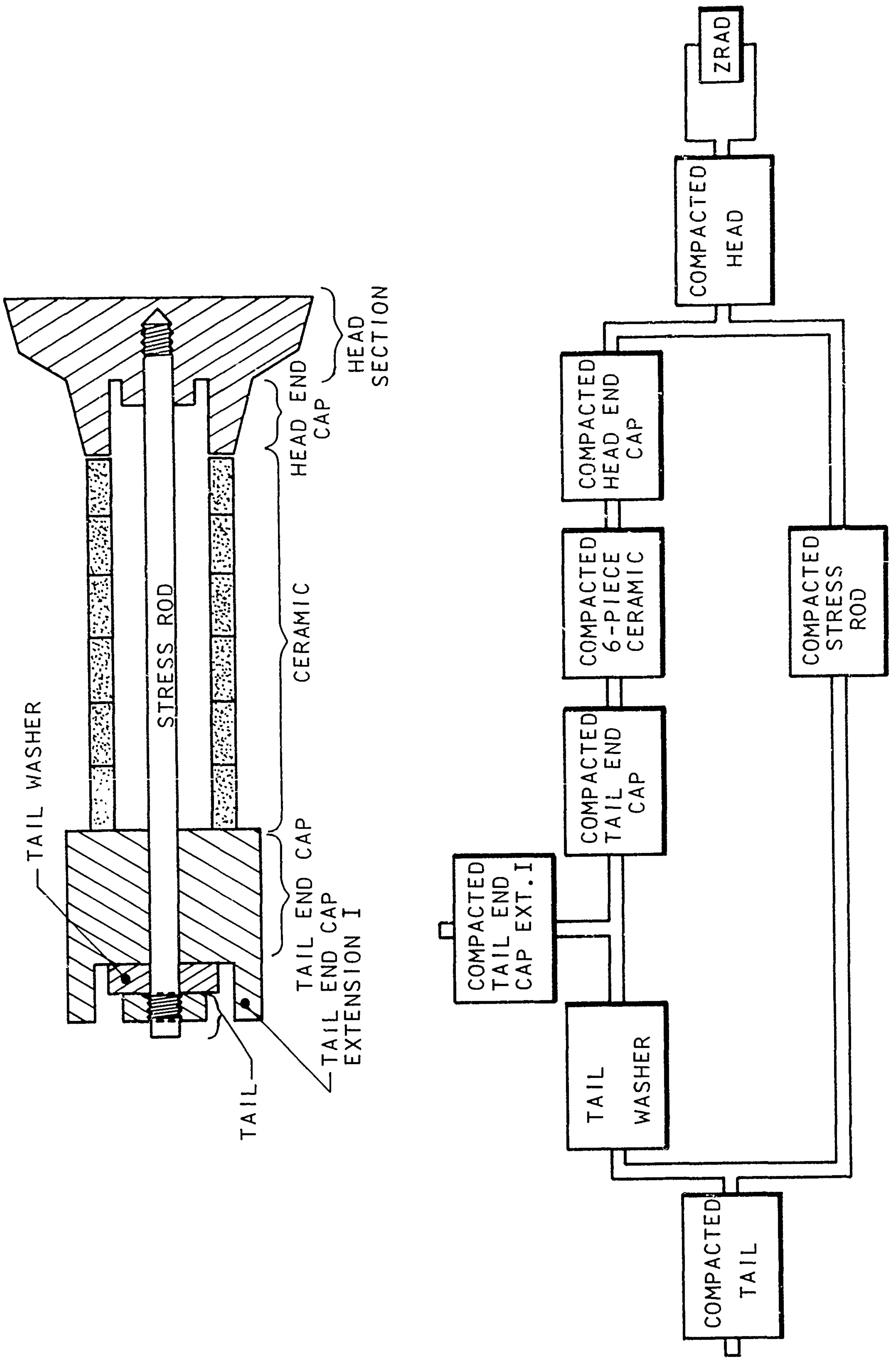

FIG. 2 - CROSS SECTION VIEW OF TRANSDUCER ELEMENT AND FOUR TERMINAL EQUIVALENT CIRCUIT

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of the element model and an accompanying equivalent circuit are shown in detail in Ref. 1.

For this study, an element model similar to the one shown in Fig. 2 and analyzed in Ref. 1 was adopted. This element has a Z_{oc} which varies with frequency as shown in Fig. 3. (Z_{oc} is plotted as a function of ka, where k is the wave number and a is the radius of the sphere). For a given ka, Z_{oc} is obtained from this curve, the constant γ is computed, and the Z_{ij} (also a function of ka) are computed. These quantities, together with the prescribed electrical signals, are then used to compute a set of head velocities $\{v_i\}$ from Eq. (6).

The final step in the computations is the evaluation of the radiation characteristics of the array. First, the pressure fields of the individual elements are computed from Eq. (7) (nearfield) or Eq. (8) (farfield). These are then superimposed to obtain the pressure field for the array. A simple angle transformation is used in the superposition, as discussed in the Appendix. This superposition of the element pressure fields completes the series of computations.

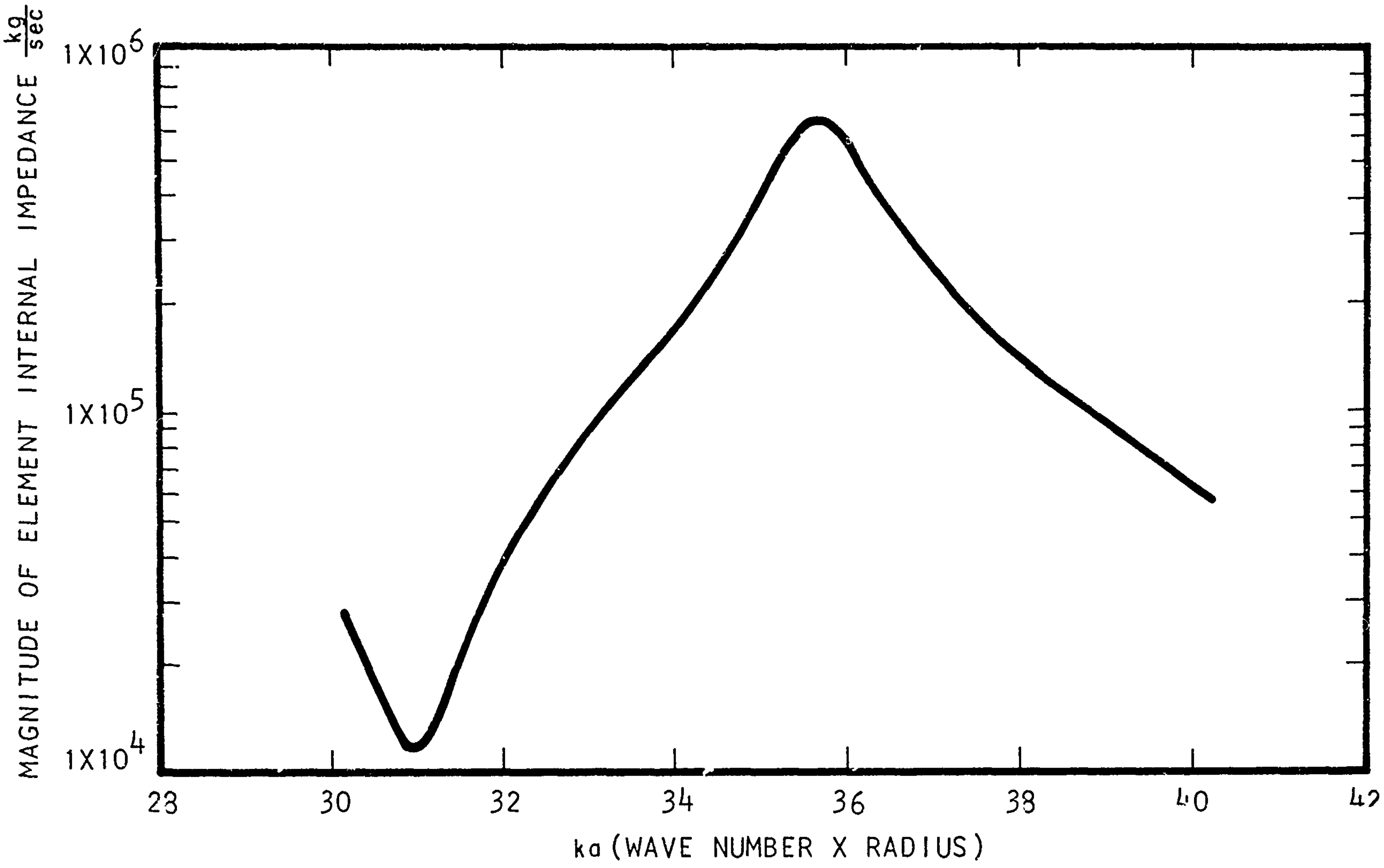

FIG. 3 MAGNITUDE OF ELEMENT INTERNAL IMPEDANCE (ZOC) VS ka OF ARRAY

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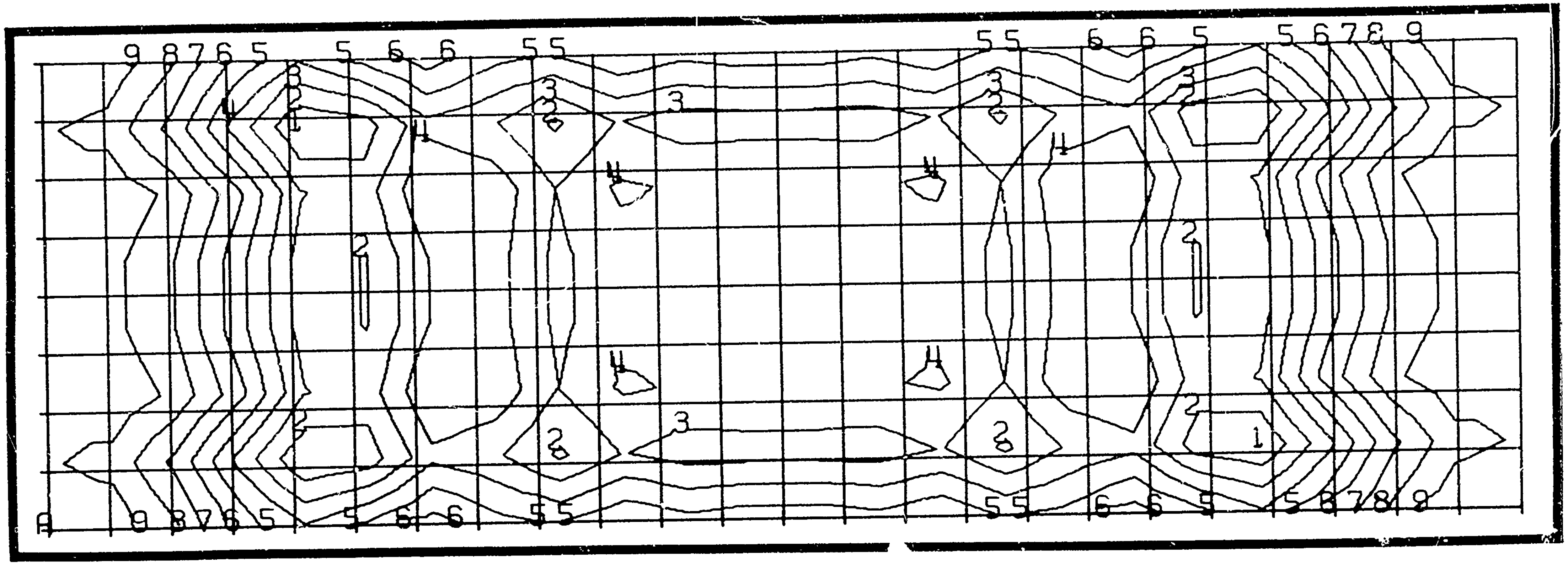
4. NUMERICAL RESULTS

Numerical Results are presented for three cases; (1) at the "tuned" frequency f_o, for ka = 36.0, where Z_{oc} is a maximum, (2) at a frequency f_L, below f_o, at which ka = 31.9, and (3) for "ideal" head velocities, i.e., the head velocity of each element is affected only by the self-impedance of that element. Results are shown both with and without dome for each of the three cases.

The transducer used in the computations consists of an array of 8 x 24 half-wavelength (at f_o, ka = 36.0), close-packed circular elements. The dome has kb = 51.0 and kh = 0.1164 at f_o and has a specific gravity of 7.86 (steel). The input currents $\{I_i\}$ are of equal unit amplitude and phase delayed to a plane for each frequency.

Figures 4-7 show contour maps of the magnitude of the total radiation loading on the array. The data in each curve are normalized within that data set. The range of loading from the minimum to maximum is divided into equal increments with symbol "1" corresponding to the lowest value and "A" corresponding to the highest value. The effect of a dome on radiation loadings is graphically illustrated by a comparison of Figs. 6 and 7. For frequency f_L and no dome, the radiation loadings are nearly constant over the broad, central portion of the array, and increase smoothly to maximum values at the corners. However, the addition of a dome (Fig. 7) causes a large buildup of radiation loading in the central portion of the array. Note that the radiation loading exceeds Z_{oc} by an order of magnitude for the case shown in Fig. 7.

The normalized magnitudes of the velocities at frequency f_L, with a dome, for two rows of the array are shown in Fig. 8. Comparing Fig. 7 and Fig. 8 shows good correlation between them, in that elements with less radiation loading have higher velocity magnitudes.



CONTOUR SYMBOL	CONTOUR VALUE ($\times 10^4$ KG./SEC.)	CONTOUR SYMBOL	CONTOUR VALUE ($\times 10^4$ KG./SEC.)
1	3.76	6	6.60
2	4.33	7	7.17
3	4.89	8	7.73
4	5.46	9	8.30
5	6.03	A	8.87

FIG. 4 CONTOUR PLOT OF THE MAGNITUDE OF THE TOTAL ACOUSTIC RADIATION LOADINGS ACROSS THE FACE OF AN 8 X 24 SPHERICAL ARRAY WITHOUT A DOME. FREQUENCY = f_0.

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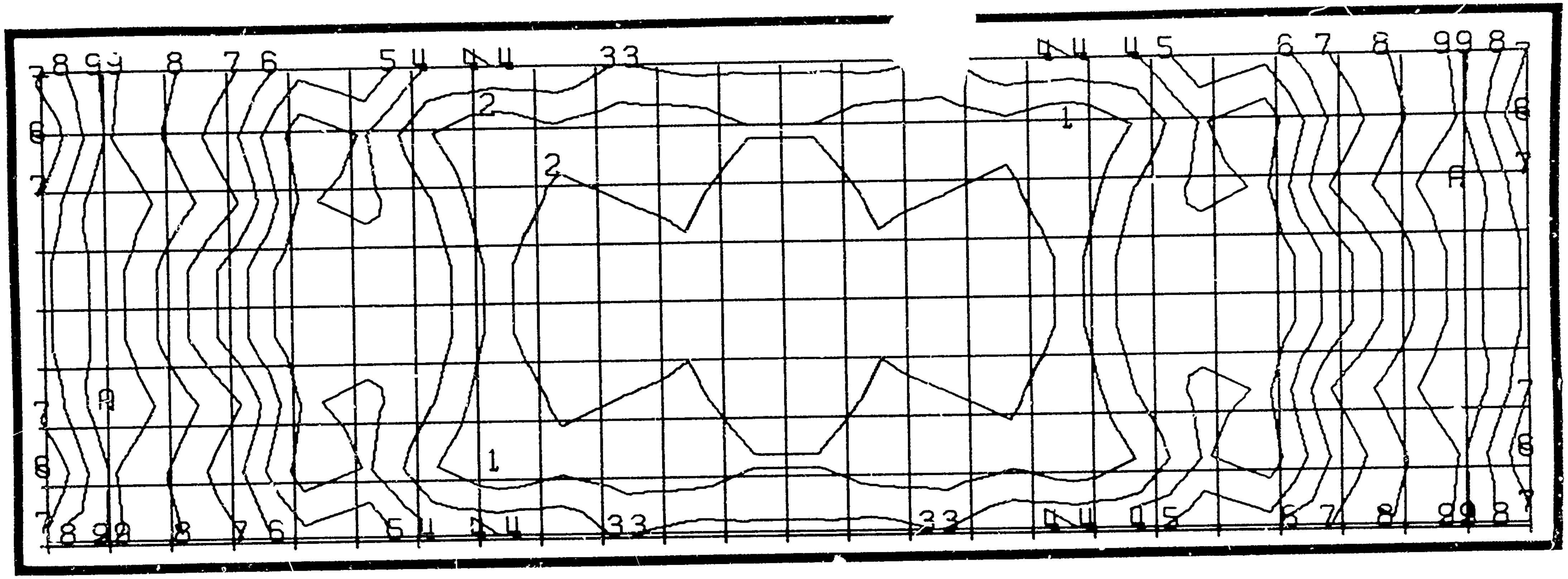

CONTOUR SYMBOL	CONTOUR VALUE ($\times 10^4$ KG./SEC.)	CONTOUR SYMBOL	CONTOUR VALUE ($\times 10^4$ KG./SEC.)
1	2.02	6	7.32
2	3.08	7	8.39
3	4.14	8	9.45
4	5.20	9	10.51
5	6.26	A	11.57

FIG. 5 CONTOUR PLOT OF THE MAGNITUDE OF THE TOTAL ACOUSTIC RADIATION LOADINGS ACROSS THE FACE OF AN 8 X 24 SPHERICAL ARRAY. DOME INTERACTIONS INCLUDED. FREQUENCY = f_0.

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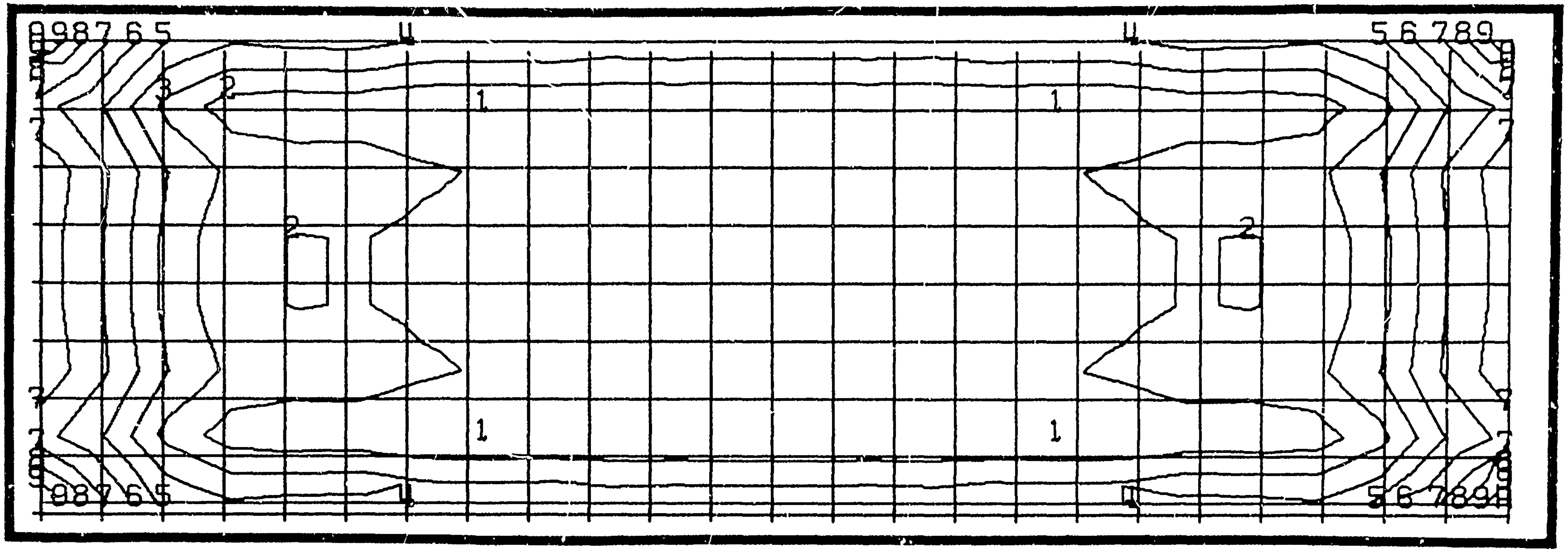

CONTOUR SYMBOL	CONTOUR VALUE ($X 10^4$ KG./SEC.)	CONTOUR SYMBOL	CONTOUR VALUE ($X 10^4$ KG./SEC.)
1	4.48	6	7.09
2	5.00	7	7.62
3	5.53	8	8.14
4	6.05	9	8.66
5	6.57	A	9.19

FIG. 6 CONTOUR PLOT OF THE MAGNITUDE OF THE TOTAL ACOUSTIC RADIATION LOADINGS ACROSS THE FACE OF AN 8 X 24 SPHERICAL ARRAY WITHOUT A DOME. FREQUENCY = f_L.

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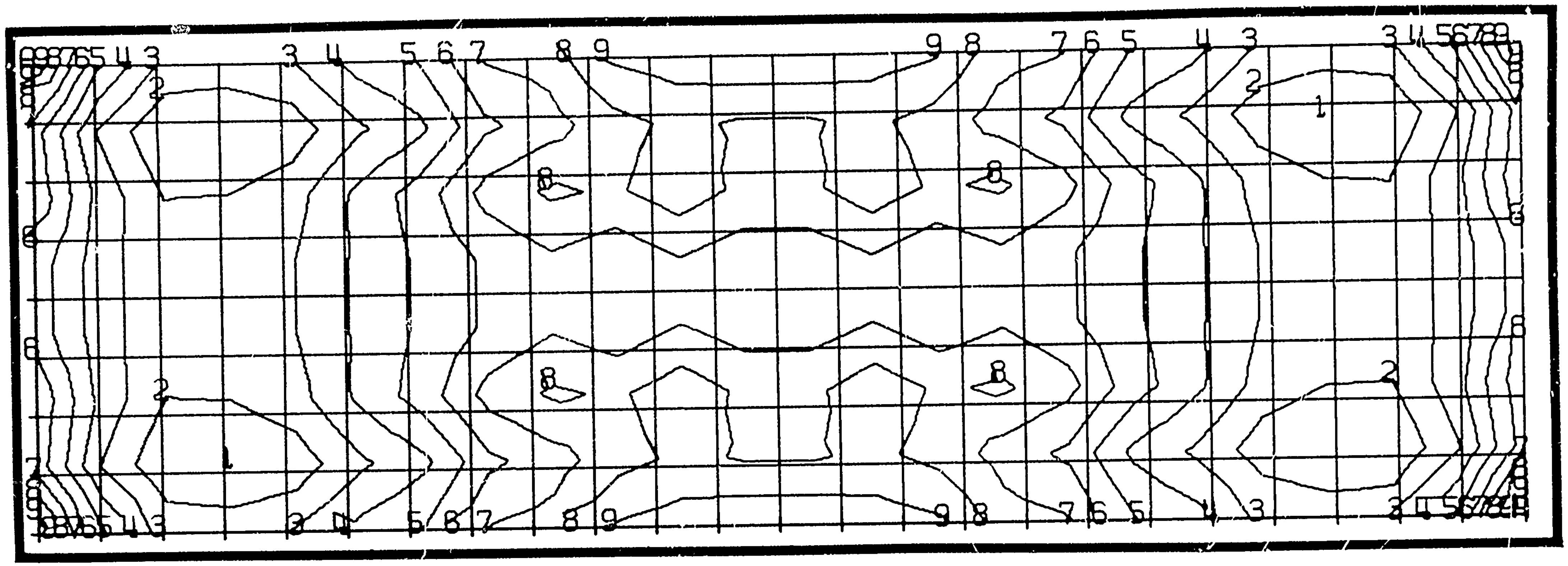

CONTOUR SYMBOL	CONTOUR VALUE ($\times 10^4$ KG./SEC.)	CONTOUR SYMBOL	CONTOUR VALUE ($\times 10^4$ KG./SEC.)
1	3.16	6	6.57
2	3.84	7	7.25
3	4.52	8	7.93
4	5.20	9	8.61
5	5.89	A	9.29

FIG. 7 CONTOUR PLOT OF THE MAGNITUDE OF THE TOTAL ACOUSTIC RADIATION LOADINGS ACROSS THE FACE OF AN 8 X 24 SPHERICAL ARRAY. DOME INTERACTIONS INCLUDED. FREQUENCY = f_L.

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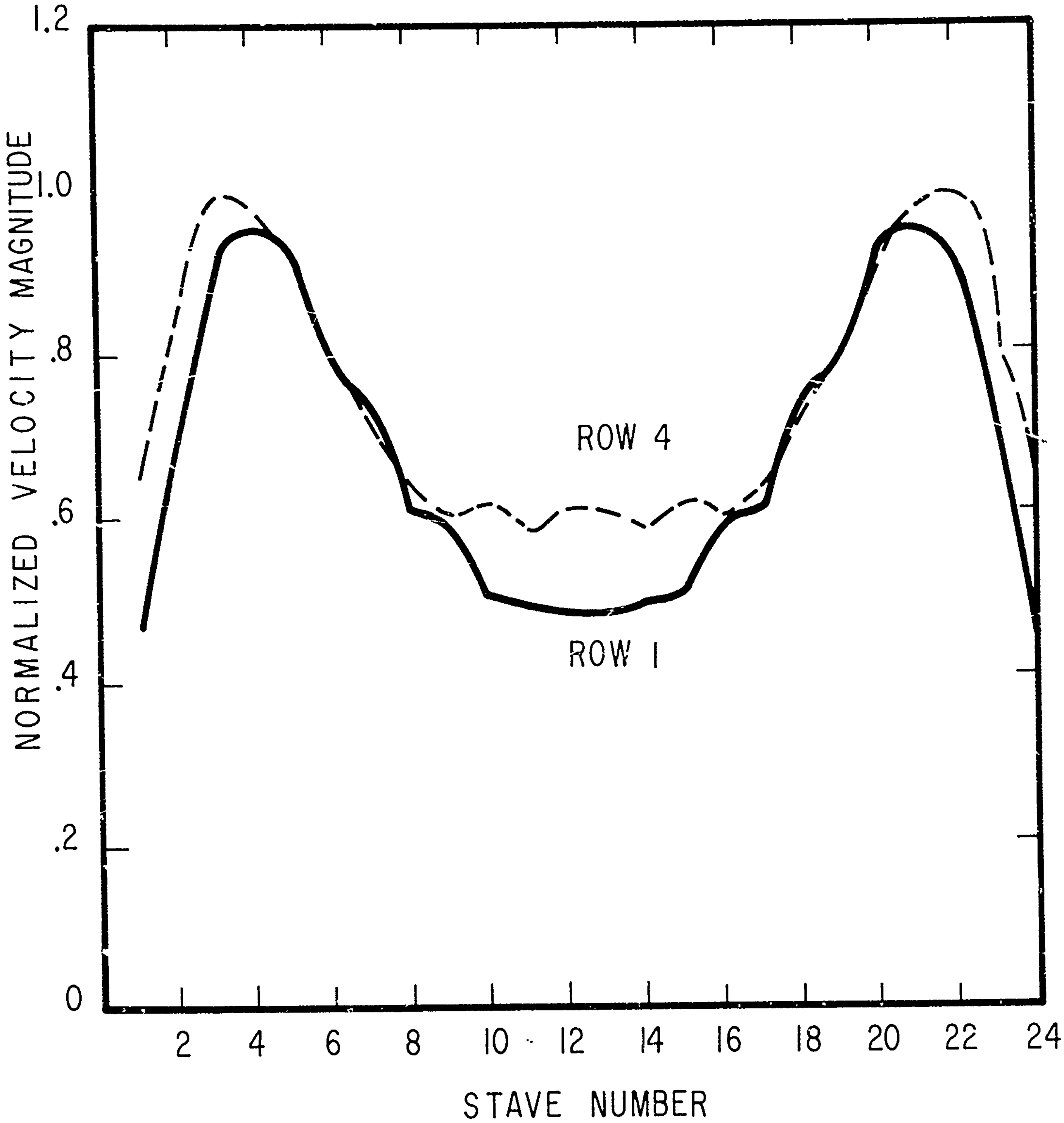

FIG. 8 NORMALIZED VELOCITY MAGNITUDE OF TRANSDUCER ELEMENT VS ELEMENT POSITION. DOME INCLUDED. FREQUENCY= f_L

—— ROW I
- - - - ROW 4

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The farfield beam patterns are presented in Figs. 9-11 with patterns for both ideal and computed velocities superimposed on the same figure. The beam patterns are normalized and do not show the effects of acoustic coupling on insertion loss. These patterns show that including element interaction results here in a slightly narrower main lobe and increased side lobe levels. The most serious difference is the case (Fig. 11) with frequency f_L, with a dome, where the first side lobe is up nearly 10 dB from the ideal case, and the main lobe is about 2° narrower. Figure 12 shows the insertion loss (change in intensity on the beam axis) for each beam pattern computed at f_o relative to the no-dome case with "ideal" velocities at f_o. It can be seen that both dome interaction and element interaction influence insertion loss. The relative importance of dome-transducer interactions and element interactions in determining insertion loss should not be generalized from these results because of the strong dependence of these interactions on dome-transducer geometry, frequency, and element size and spacing.

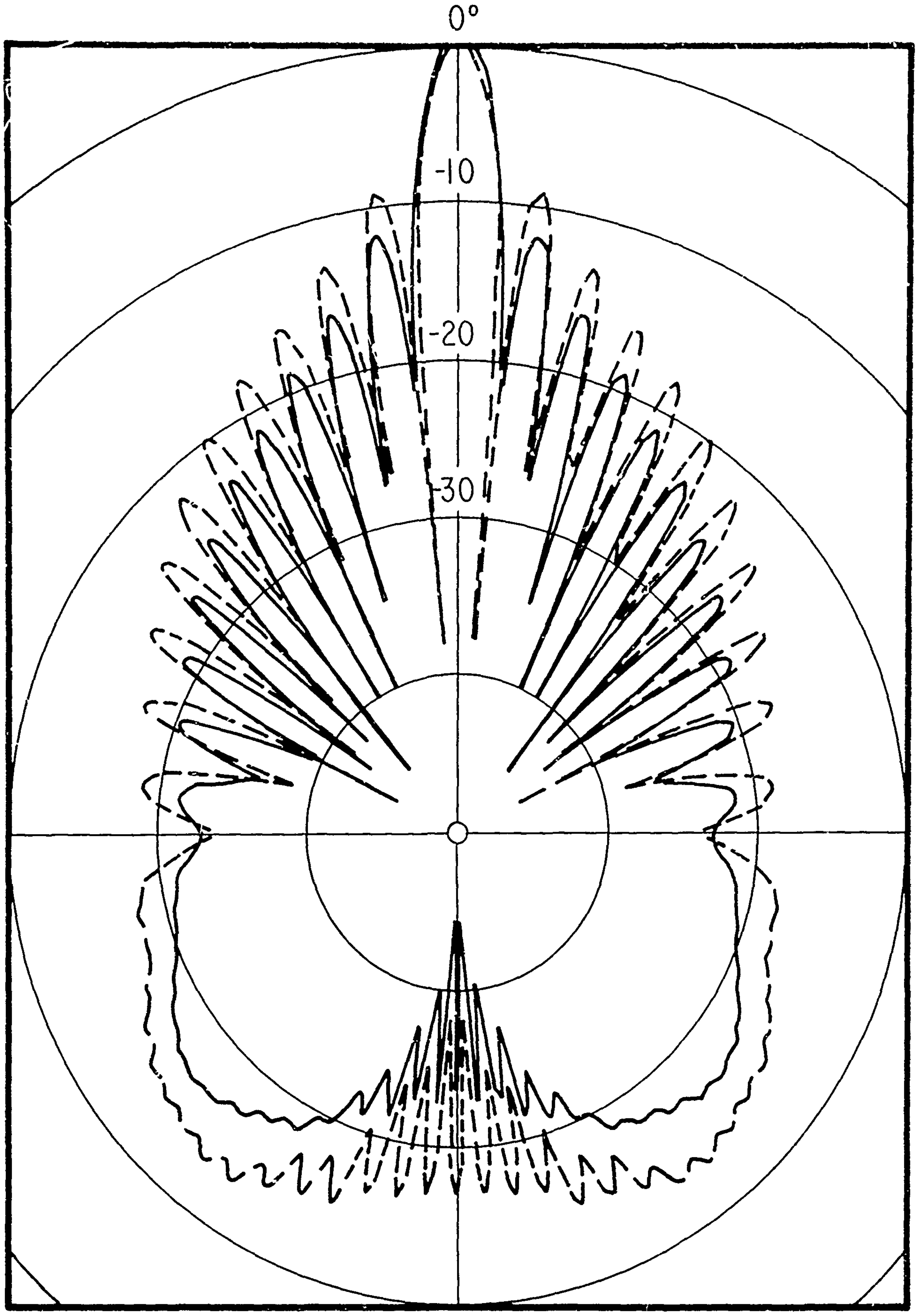

FIG. 9 AZIMUTHAL BEAM PATTERN FOR BARE TRANSDUCER

FREQUENCY = f_0

——— IDEAL VELOCITIES

- - - - COMPUTED VELOCITIES

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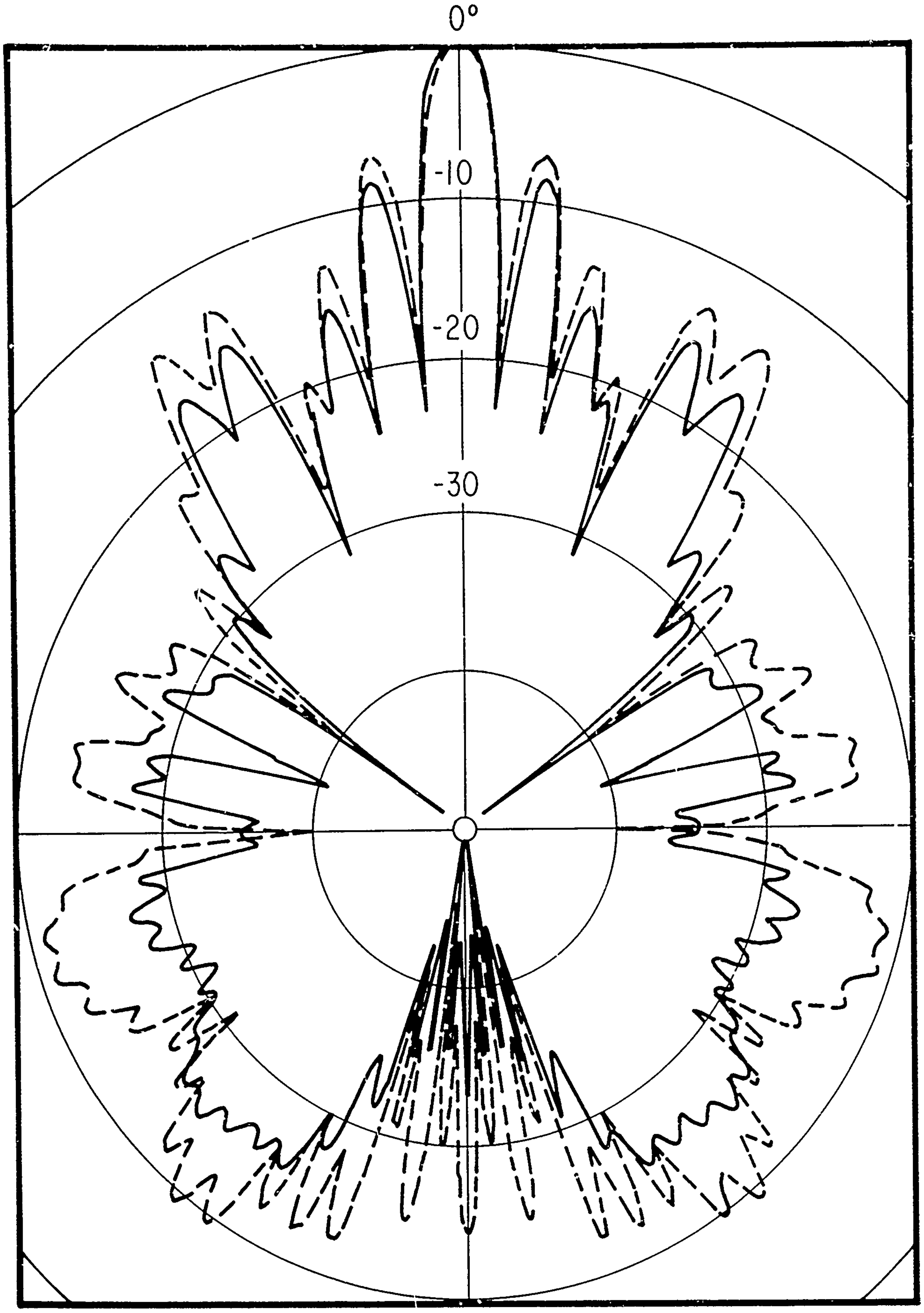

FIG. 10 AZIMUTHAL BEAM PATTERN FOR DOME/TRANSDUCER.

FREQUENCY = f_0

——— IDEAL VELOCITIES

- - - - - COMPUTED VELOCITIES

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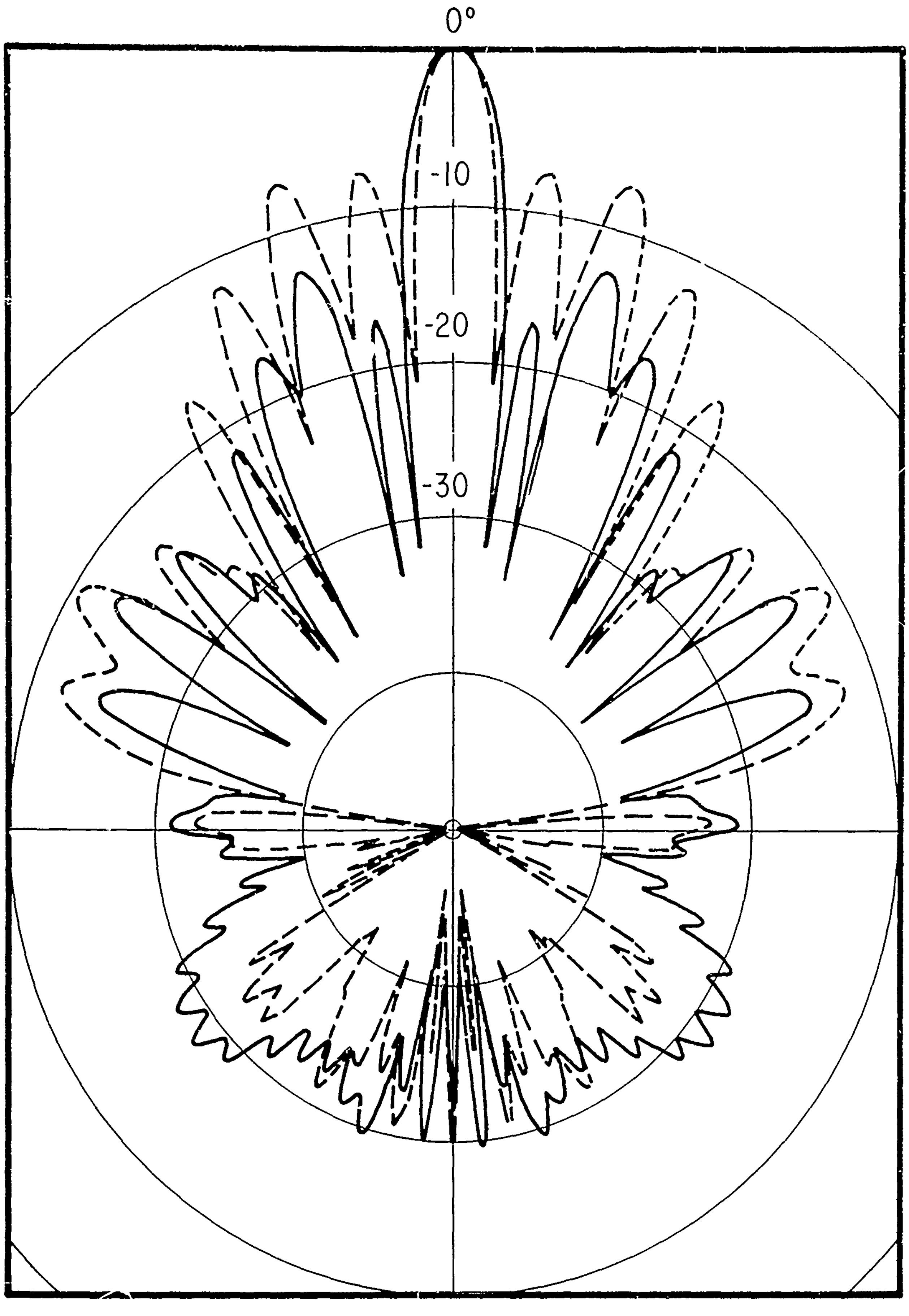

FIG.11 AZIMUTHAL BEAM PATTERN FOR DOME / TRANSDUCER

FREQUENCY= f_L

———IDEAL VELOCITIES

------COMPUTED VELOCITIES

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FREQUENCY	RELATIVE SOURCE LEVEL (DB)			
	BARE TRANSDUCER, IDEAL VELOCITIES	DOME/TRANSDUCER IDEAL VELOCITIES	BARE TRANSDUCER, COMPUTED VELOCITIES	DOME/TRANSDUCER, COMPUTED VELOCITIES
f_o	0.0	-1.2	-4.6	-5.7

FIG. 12 INSERTION LOSS RELATIVE TO BARE TRANSDUCER WITH IDEAL VELOCITIES AND FREQUENCY f_o

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5. CONCLUSIONS

This memorandum demonstrates a procedure for analyzing the effects of transducer element interactions and dome/array interactions on element response and, consequently, radiation characteristics of the array. As shown in the computed results, these interactions can have significant effect on the performance of an array. Realistic analysis of interaction effects is particularly important if the array must perform over a frequency band. In this case the internal impedance of the element, usually a maximum over a narrow range of frequency, and the radiation impedance can be of similar magnitude at the upper and lower limits of the operational frequency band. As a result, the radiation characteristics of the array can be degraded.

6. REFERENCES

1. Notes for U. S. Navy Electronics Laboratory Seminar on Transducer Array Analysis and Evaluation, (Rough Draft), 14 June 1965.

2. "Summary Report - Exploratory Development Dome Studies," Vol. I and II, TRACOR Document Number 67-917-U, October 9, 1967.

3. Douglass, R. E. and B. E. Jay, "Radiation Characteristics of a Spherical Transducer Surrounded by a Concentric Shell," TRACOR Document Number 67-178-U, February 10, 1968.

APPENDIX
SPHERICAL TRANSDUCER-DOME MODEL

The analysis of the spherical transducer with concentric spherical dome, outlined here, is presented in a previous technical memorandum [Ref. 3]. The geometry of the spherical transducer and dome is shown in Fig. 1.

The first step in the analysis is to obtain the solution for the sound pressure radiated by a single circular piston. A piston vibrating with harmonic time dependence will produce a sound pressure field between the dome and transducer which can be represented by the expansion (in which $e^{-i\omega t}$ time dependence is understood),

$$p_1 = \sum_{m=0}^{\infty} \left[A_m j_m(kr) + B_m n_m(kr)\right] P_m(\cos \psi). \qquad \text{(A-1)}$$

This form is standard for solutions of the wave equation in spherical coordinates (r, ψ) if there is symmetry about the axis from which the angle ψ is measured. The P_m are Legendre polynomials of argument $\cos \psi$ and the j_m and n_m are, respectively, the spherical Bessel and Neumann functions of argument kr. The A_m and B_m are constants which can be adjusted to make the solution form fit whatever boundary conditions are prescribed on the transducer and shell surfaces. The boundary conditions arise in equating the fluid and solid velocity components normal to the fluid-solid interfaces. Velocity components tangential to the interfaces are not necessarily equal since the fluid is assumed inviscid.

For the case at hand, the fluid velocity is equal to the piston velocity on the piston surface and is zero on the remaining transducer surface. The piston velocity, $v_o e^{-i\omega t}$, will be considered to be purely radial, so the boundary condition at the transducer surface $r = a$ is

$$\frac{k}{i\omega\rho} \sum_{m=0}^{\infty} [A_m j_m'(ka) + B_m n_m'(ka)] \, P_m(\cos\psi) = \begin{cases} v_o, & 0 \le \psi < \alpha \\ 0, & \alpha < \psi \le \pi \end{cases} . \quad \text{(A-2)}$$

Here the angular extent of the piston is $0 \le \psi < \alpha$, the radial fluid velocity component being zero outside of this range. The primes on the j_m and n_m denote differentiation with respect to argument.

While the pressure field between the transducer and dome has the form given in Eq. (A-2), the sound pressure field, p_2, outside the dome must have a form which describes only outgoing radiation. This form is

$$p_2 = \sum_{m=0}^{\infty} C_m h_m(kr) \, P_m(\cos\psi) \quad , \quad \text{(A-3)}$$

where $h_m = j_m + i\, n_m$ is a spherical Hankel function and the C_m are determined by the boundary condition at the outer dome surface. Since the dome is thin relative to an acoustic wavelength, it is sufficient to apply the velocity continuity condition at the dome midsurface, $r = b$, rather than at the actual fluid-dome interfaces. Thus,

$$\frac{1}{i\omega\rho} \frac{\partial p_1}{\partial r}\bigg|_{r=b} = \frac{1}{i\omega\rho} \frac{\partial p_2}{\partial r}\bigg|_{r=b} = w \quad , \quad \text{(A-4)}$$

w being the radial velocity distribution of the dome. From the given boundary conditions and the analysis of the dome motion given in the earlier technical memorandum [Ref. 3] the unknown coefficients A_m, B_m, and C_m can be found. Equations (A-1) and (A-3) can now be used to find nearfield and farfield pressures respectively.

The pressure field, p_1, generated by a single piston and evaluated at the transducer surface, is found to be

$$p_1 = V_o \frac{i\rho c}{2} \sum_n \Gamma_n [P_{n-1}(\cos\alpha) - P_{n+1}(\cos\alpha)]\, P_n(\cos\psi) \quad , \qquad \text{(A-5)}$$

where

$$\Gamma_n = \frac{h_n(ka) + \frac{Z_n}{i\rho c}(kb)^2 h_n(kb)[j_n(ka)n_n'(kb) - j_n'(kb)n_n(ka)]}{h_n'(ka) + \frac{Z_n}{i\rho c}(kb)^2 h_n'(kb)[j_n'(ka)n_n'(kb) - j_n'(kb)n_n'(ka)]} ,$$

and Z_n is the modal impedance of the shell (see Ref. 3). This pressure field causes a net force on the other elements in an array.

The farfield pressure distribution can be computed from Eq. (A-3) by substituting into it the asymptotic form of $h_m(kr)$ for large r, which is

$$\lim_{kr \to \infty} h_m(kr) = \frac{i^{-m-1}}{kr} e^{ikr} \quad .$$

Thus,

$$\lim_{r \to \infty} p(r,\psi) = \rho c v_o \frac{e^{i(kr-\omega t)}}{r} F(\cos\psi) \quad , \qquad \text{(A-6)}$$

where

$$F(\cos\psi) = \frac{1}{2k} \sum_{m=0}^{\infty} \frac{i^{-m}[P_{m-1}(\cos\alpha) - P_{m+1}(\cos\alpha)]\, P_m(\cos\psi)}{h_m'(ka) + \frac{Z_m}{i\rho c}(kb)^2 h_m'(kb)[j_m'(ka)n_m'(kb) - j_m'(kb)n_m'(ka)]} .$$

This expression determines the farfield characteristics for a single piston.

The farfield (or, similarly, the nearfield) sound pressure, $p(r,\theta,\Phi)$, of an array of pistons can be found by superposition of the fields produced by the individual pistons. The piston with center at θ_J, Φ_J, ψ_J on the transducer has velocity $v_J e^{-i\omega t}$. The cosine of the angle between θ, Φ and θ_J, Φ_J is just

$$\cos \psi_J = \cos \theta \cos \theta_J + \sin \theta \sin \theta_J \cos (\Phi - \Phi_J) \quad . \qquad \text{(A-7)}$$

Superposition of the pressure fields of an array of N pistons gives

$$p(r,\theta,\Phi) = \rho c \frac{e^{i(kr - \omega t)}}{r} D(\theta,\Phi) \quad , \qquad \text{(A-8)}$$

where

$$D(\theta,\Phi) = \sum_{J=1}^{N} v_J F(\cos \psi_J) \quad .$$

The farfield intensity distribution I is $\frac{1}{2\rho c} p^* p$, where the asterisk denotes the complex conjugate. Therefore,

$$I = \frac{\rho c}{2} \frac{1}{r^2} |D(\theta,\Phi)|^2 \quad . \qquad \text{(A-9)}$$

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13 ABSTRACT

A procedure is described which permits the computation of the radiation characteristics of a transducer array given the electrical inputs to the transducer elements. The effects of transducer element interactions (acoustic coupling) and dome-transducer interactions are included in the computation.

The dome-transducer configuration chosen for this study is a rectangular array of circular elements mounted in a spherical baffle. This array is surrounded by a concentric, homogeneous dome. Numerical results include (1) radiation loadings on the transducer elements, (2) response (head velocity) of each element to prescribed electrical inputs and (3) farfield beam patterns for the dome/array. The results indicate that element interactions and dome/array interactions can affect element response to the electrical input and thereby degrade transmit performance of the transducer array.

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